

Elasticity theory of structuring

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We reinforce the foundations of our quantitative approach to structuring by considering a large class of rational investors. Again, the Bayesian laws of information processing provide us with simple yet powerful tools. Structuring of investment derivatives is summarized as a manufacturing process. This allows for performance, quality and safety to be built into the product at the level of individual production stages – just as it is done in the established manufacturing industries.

1 Introduction

In [1] we reviewed the shortcomings of the pre-crisis approach to product design, advocated the need for a better more quantitative approach and proposed practical solutions. So far, most of our examples have been centered around the important special case of the growth-optimizing investor, i.e. investor which seeks maximum expected rate of return. This gave us the range in which most sensible investors must be – between risk-free and growth-optimizing [2]. In this paper we extend our framework to individual realistic investors.

2 Investor equivalence principle

It turns out that understanding growth-optimizing investor opens a door for understanding a much larger class of investors. We already mentioned and even used this fact in [2]. In this section we reveal the basic logic behind this fact and by doing so prepare the ground for more detailed explorations of the subsequent sections.

For the growth-optimizing investor we showed that investor's belief, $b(x)$, regarding future values of a market variable, x , is related to the market-implied view, $m(x)$, via the Bayes theorem where the role of the likelihood function is played by the growth-optimal payoff, $f(x)$. Mathematically,

$$b(x) = f(x) m(x). \quad (1)$$

In order to consider the case of a general investor, we start by revisiting the arguments which lead us to Eq. (1) and repeat them to the point before we make any assumptions

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regarding investor's goals. Following [2], we start by partitioning the range of possible values of the market variable x into non-overlapping intervals using a discrete mesh $(\dots, x_i, x_{i+1}, \dots)$. Let R_i be the return offered by the market on a digital spread which pays either 1 or zero depending on whether or not x falls between x_i and x_{i+1} . In [2] we argued that for a very large class of investors the problem of optimal investment is equivalent to optimal splitting of capital across the digital spreads. Let $\{\beta_i\}$ be the proportions in which the investor decides to partition their investment ($\sum_i \beta_i = 1$). Only one digital spread can mature in the money, so for the payoff, F , from the investment we compute

$$F_k = \beta_k R_k, \quad (2)$$

where k is the index of the spread that matures in the money. Defining $m_k = 1/R_k$, rearranging the terms and using the notation of Eq. (1) we derive

$$\beta(x) = F(x) m(x). \quad (3)$$

For the sake of completeness we remind the reader that we do not want to write trivial normalization constants, so both Eq. (1) and Eq. (3) are written as if $\sum_k R_k^{-1} = 1$ – the convenient case when the normalization of $m(x)$ is automatically guaranteed. Readers which are not comfortable with this convention or those who would like to examine more general settings may want to review the appendix in Ref. [2].

Above derivation covers all investment scenarios which can be broken down into mutually exclusive events. Given that the choice of the underlying variable remains completely up to the investor this is a very general setting. Indeed, x can be pretty much anything: from bespoke stock portfolios, derivative prices and proprietary volatility indices to temperature readings in Texas. The choice of $\beta(x)$ in (3) is also pretty general – it doesn't even need to be rational in any way. Comparison of Eqs. (1) and (3) leads us to a somewhat surprising conclusion which, in view of its generality, we formulate below as a principle.

Any investor can be viewed as growth-optimizing.

Indeed, as we see from Eq. (1) and Eq. (3), general investors choose the same product, i.e. behave in the same way, as a growth-optimizing investor whose belief $b(x)$ happens to coincide with $\beta(x)$.

3 Logical investors

In the previous section we discovered that growth-optimizing investors are surprisingly versatile. In fact, it looks like they can be educated to act as if they had sophisticated risk preferences. Understanding this education process mathematically is key to making the investor equivalence principle into a practical tool. To this end we narrow the scope of our investigations and focus on the case of rational investors.

It can be shown that rational investors behave as if they were maximizing the expected value of utility [3]. The expected utility approach which follows from this observation is well known in economic theory. We will use it here as well. It is important, however, to emphasize that by adopting the expected utility approach we do not reduce the generality

of our arguments in the same way as it happens in economic theories. Indeed, the concept of rational investors appears in economics as a strong assumption which often leads to erroneous conclusions. In the theory of product design the situation is quite different. For us rationality is not an assumption. It is part of the goal. We understand that being rational is not easy, so we want tools that can help.

Consider a rational investor with utility $u()$ and a view on x given by the probabilities $\{b_i\}$, where each b_i measures the degree of investor's belief for x to end up between x_i and x_{i+1} . Because logarithm is a monotonically increasing function, we can, without further loss of generality, write the utility as a function of the logarithmic rate of return. We also allow the utility to depend on x explicitly. The problem of optimal investment is solved by maximizing the expectation

$$\sum_i b_i u\left(\ln(\beta_i R_i), x_i\right) \quad (4)$$

over all possible proportions $\{\beta_i\}$ subject to the constraint $\sum_i \beta_i = 1$. The Lagrangian for this optimization reads

$$\mathcal{L}(\{\beta_i\}, \lambda) = \sum_i b_i u\left(\ln(\beta_i R_i), x_i\right) + \lambda \left(\sum_i \beta_i - 1\right). \quad (5)$$

By setting $\partial \mathcal{L} / \partial \beta_k = 0$ we compute

$$b_k u'\left(\ln(\beta_k R_k), x_k\right) = -\lambda \beta_k, \quad (6)$$

where prime denotes derivative with respect to the first argument. This implies

$$\sum_i b_i u'\left(\ln(\beta_i R_i), x_i\right) = -\lambda \sum_i \beta_i = -\lambda. \quad (7)$$

Substituting this back into Eq. (6) we obtain

$$\beta_k = \frac{u'\left(\ln(\beta_k R_k), x_k\right) b_k}{\sum_i u'\left(\ln(\beta_i R_i), x_i\right) b_i}. \quad (8)$$

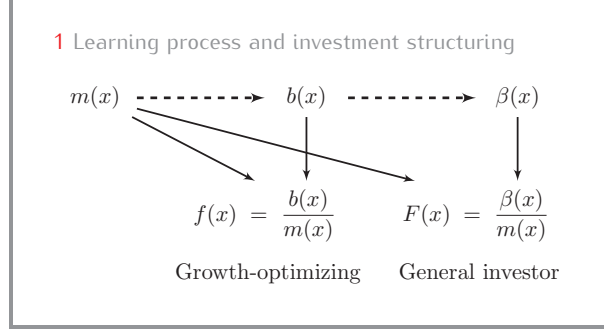
This equation makes sense only for positive β_k . This is indeed guaranteed by the monotonicity of the utility function with respect to return $u' > 0$ (the limiting case of zero β_k is trivial and can be removed from the optimization problem (5)). Solving this equation for β_k gives us the optimal investment strategy. This includes the growth-optimizing investor as a special case. Indeed, in this case we compute $u' = 1$, Eq. (8) reduces to the Kelly equation, $\beta_k = b_k$, which in view of (3), leads to our basic equation (1).

On the purely technical level, the optimization problem (4) as well as its solution (8) can be viewed as trivial generalizations of the Kelly argument [4]. A fundamentally important insight, however, can be gained by using this simple generalization to study the learning process which supports the investment.

Substituting Eq. (2) into (8) and writing the result in the notation of Eq. (1) we get

$$\beta(x) = \frac{u'\left(\ln F(x), x\right)}{\int u'\left(\ln F(\tilde{x}), \tilde{x}\right) b(\tilde{x}) d\tilde{x}} b(x). \quad (9)$$

Let us now use the investor equivalence principle and look at this formula through the eyes of a growth-optimizing investor. As always, the investor begins by examining the market, $m(x)$. Having done some research, the investor formulates his belief, $b(x)$. Growth-optimizing investor would normally stop here and proceed directly to the construction of the payoff via equation (1), as shown on Figure 1. What would it take for the growth-optimizing investor to change his mind and use $\beta(x)$ instead of $b(x)$? There is only one thing that can do that – it is information. The information must of course be new and relevant to the investment. Upon receipt of such information, the growth-optimizing investor would have to take his belief $b(x)$ and update it once again using Bayes theorem arriving at $\beta(x)$. Equation (9) describes exactly this additional learning step.



What information is learned during this additional step? Clearly it has something to do with the personal preferences of the general investor. Not surprisingly, the likelihood function describing this learning step depends on the utility function. Mathematically the preferences are captured relative to the growth-optimizing investor – this is the reason for introducing our slightly non-standard notation for utility as a function of logarithmic return. Personal preferences can have very different underlying reasons, but whatever they are, Eq. (1) tells us how they can be learned (in principle) by the growth-optimizing investor.

Incorporating all information that is relevant to investments (objective or personal) is clearly an important ability for an investor. Those investors which can do that without contradicting Bayes theorem are especially important, so we give them a name. Let us call them *logical investors*. In the Appendix we provide additional context both to justify this name and to give a better feel for the role such investors play in our theory.

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4 Payoff elasticity equation

Dividing both sides of Eq. (9) by $m(x)$ and using equations (1) and (3) we derive

$$F(x) = \frac{u'(\ln F(x), x)}{\int u'(\ln F(\tilde{x}), \tilde{x}) b(\tilde{x}) d\tilde{x}} f(x). \quad (10)$$

This equation can be viewed as a mathematical expression of the investor equivalence principle. It should allow us to compute the general payoff profile, $F(x)$, from the corresponding growth-optimal profile, $f(x)$. This equation, however, has a couple of technical issues which affect its usefulness in practical applications. First, it uses the notion of utility. Second, it is an integral equation for F . In this section we discuss and address both of these issues.

It is well known that affine transformations of utility do not have any effect on the investors' preferences. To put it simply, the concept of utility effectively ignores the two

most fundamental mathematical operations – addition and multiplication by a number. This fact makes utility functions a little bit awkward, especially when we want to talk about investors with different degrees of risk aversion. To remedy the situation, a variety of measures for risk-aversion were introduced. The most popular of them are probably the Arrow-Pratt measures of absolute and relative risk aversion:

$$A(F) = -\frac{U''(F)}{U'(F)}, \quad R(F) = -F \frac{U''(F)}{U'(F)}, \quad (11)$$

where $U(F(x)) = u(\ln F(x), x)$ is the standard definition of utility. We would like to transform Eq. (10) so that the concept of utility appeared only via a meaningful risk-aversion quantity. Note that the classic utility, U , does not depend on x explicitly. In this case we can write $u(\ln F, x) = u(\ln F)$. We investigate this case first and then discuss what happens in general.

We want to convert integral equation (10) into a more manageable differential form. The concept of elasticity gives us a particular way of differentiating that proved to be especially useful in economics and finance. Elasticity of a function, $\phi(x)$, with respect to its argument, x , is defined as the derivative

$$\frac{d \ln \phi(x)}{d \ln x}. \quad (12)$$

This measures the percentage change in the function's value with respect to percentage change in its argument. Taking the logarithm on both sides of Eq. (10), forgetting temporarily about the explicit dependence of u on x and differentiating we obtain

$$d \ln F = \frac{1}{u'(\ln F)} u''(\ln F) d \ln F + d \ln f. \quad (13)$$

Rearranging the terms,

$$\frac{d \ln F}{d \ln f} = \frac{u'(\ln F)}{u'(\ln F) - u''(\ln F)}. \quad (14)$$

By direct calculation we have

$$U'(F) = \frac{u'(\ln F)}{F}, \quad U''(F) = \frac{u''(\ln F) - u'(\ln F)}{F^2}. \quad (15)$$

Recalling the definition of relative risk aversion (11), we can now rewrite Eq. (14) as

$$\frac{d \ln F}{d \ln f} = \frac{1}{R}. \quad (16)$$

This simple equation is the central technical result of this paper. It gives us a fundamental link between payoff elasticity and risk aversion. The more risk aversion we have the less elastic is the payoff. On the practical side, this equation allows us to compute the optimal payoff F from the growth-optimal f and the risk-aversion profile R of the client. Conversely, we are now also able to compute risk-aversion profiles directly from clients' positions.

For the sake of completeness we mention some alternative forms of Eq. (16) which can be used depending on the application. Considering the payoff elasticity equation (16) for two different general investors we derive

$$\frac{d \ln F_1}{d \ln F_2} = \frac{R_2}{R_1}. \quad (17)$$

This equation shows that other payoff profiles (not necessarily growth-optimal) can serve us as building blocks. The concept of elasticity can be replaced by the ordinary differentiation if we decide to work in terms of a less fundamental measure of absolute risk aversion:

$$\frac{dF}{df} = \frac{1}{fA}, \quad \frac{dF_1}{dF_2} = \frac{A_2}{A_1}. \quad (18)$$

How would the above derivation change if we allowed for explicit dependence of u on x ? The payoff elasticity equation would stay the same but the expressions for both R and A would become more complicated. In particular they would acquire explicit dependence on the state x , but would of course reduce to the original Arrow-Pratt definitions in the special case of state-independent preferences. On the practical level, the important thing to mention is that essentially all of these equations have been solved and present no further technical challenge. Indeed, Picard-Lindelöf theorem provides the necessary theory and even offers an explicit construction of the general solution (Picard iteration method).

5 Illustrations

5.1 One-parameter investor families

By a one-parameter family we mean a set of investors whose degree of risk aversion is controlled by a single number, e.g. constant absolute or constant relative risk aversion. The usefulness of one-parameter families comes from the fact that the position of an investor within the family can be determined by asking the investor a single question regarding, for instance, their maximum acceptable loss. To illustrate this point, let us consider a particularly simple case of

$$A = \frac{a}{f}, \quad (19)$$

where a is a constant which controls the strength of risk aversion. We immediately derive

$$F = \frac{1}{a}(f - 1) + 1. \quad (20)$$

We see that a effectively scales the payoff around the bond line $F = 1$. This is exactly how Figure 2 in [2] was constructed. The value of a can be easily found by matching investors maximum acceptable loss.

5.2 Product validation

Can our theory accommodate every imaginable investor? Of course not. Just like there are physical systems which do not follow Newtonian mechanics, one can imagine investors

that demand greater flexibility than our equations would allow. As explained in the Appendix, we should be very cautious in offering assistance to such investors as not every desire is necessarily wise. At the moment it may be more prudent to focus on the many unexplored possibilities that are already offered by our theory.

Having said that, it is very important to acknowledge that investment ideas are borne in all sorts of ways and good ideas may not necessarily come through an explicit use of our theory. To recognize such cases we want the ability to verify that a given investment strategy is both rational and logical.

In order to see how this can be done, let us examine the family of investors considered in [6]. The investors are looking for a payoff structure, $h(x)$, which solves a certain optimization problem. Using our notation, the optimization problem reads:

$$\max_h \left[\underbrace{\int h(x)b(x) dx}_{\text{mean}} - \frac{R_a}{2} \underbrace{\int h^2(x)m(x) dx}_{\text{variance}} \right] \quad (21)$$

$$\text{subject to } \int h(x)m(x) dx = 0, \quad (22)$$

where R_a is a parameter that controls the degree of risk aversion. Shimko introduces this problem by analogy with the mean-variance optimization approach of Markowitz. It is important to note however that the mean and the variance, given by the first and the second term of (21) respectively, are computed using two very different distributions. In particular, the mean is computed using the investor-believed $b(x)$, while the definition of variance is using the market-implied $m(x)$. As a result, the optimization setup (21-22) does not fit well into utility maximization paradigm and so cannot be used to recommend the investment even as rational. It can however be used as an idea, an inspiration that points to a particular investment product which needs further justification.

In what follows we put aside any attempts to justify the optimization (21-22) and proceed by directly examining its solution, $h(x)$. By doing so we put ourselves in a rather typical situation when an investment product is proposed which seems clever but with possible questions regarding its real quality. Before we can recommend the product, we want to achieve two goals. First, we want to verify that $h(x)$ can be viewed as a rational strategy pursued by a logical investor. Second, we want to find and describe such an investor. This would give us the necessary level of transparency and clarity for justification of the trade. The expression for $h(x)$ is given by the first equation in [6], namely

$$h(x) = \frac{b(x) - m(x)}{m(x)} \frac{1}{R_a}. \quad (23)$$

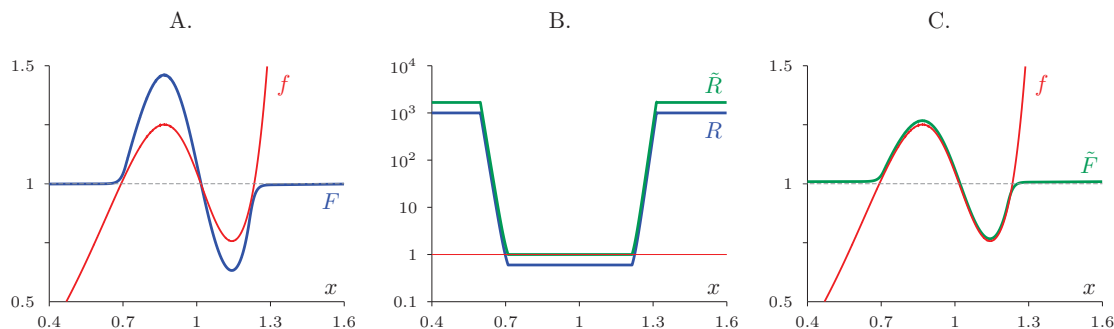
By rearranging the terms we can rewrite this equation as

$$b(x) = (R_a h(x) + 1) m(x). \quad (24)$$

This allows us to understand the investment through the eyes of a growth-optimizing investor. Indeed, using equation (1) we derive

$$R_a h(x) + 1 = f(x), \quad (25)$$

2 Implied and adjusting risk aversion



Note: Adjusting risk-aversion is an easy and intuitive way of fine-tuning exposure. Profiles on Figure A are taken from Figure 2.C of [1]. A simple change to the risk-aversion profile (Figure B) removes excess exposure exhibited by F in the ATM region (compare Figures C and A).

where $f(x)$ is the growth-optimal payoff structure. Finally, we see that

$$h(x) = \frac{1}{R_a} (f(x) - 1). \quad (26)$$

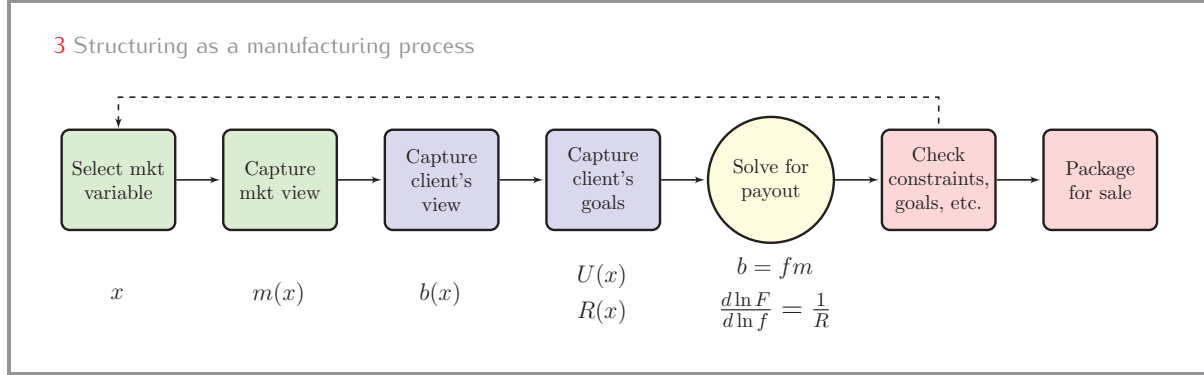
We immediately recognize this equation as a particular case of the one-parameter investor family which we considered above. Indeed, the only difference between this equation and equation (20) is the absence of an additive constant – the upfront investment cost of 1 – which drops out from (26) on the account of the constraint (22). This constraint assumes the existence of a perfectly liquid two-way market in options on x which effectively allows the investor to borrow from the market and set up the investment at zero upfront cost. Such assumptions are very important to note and check. Other than that we managed to verify that the solution of the optimization (21)-(22) can be viewed as a rational investment pursued by a logical investor with understandable risk aversion strategy.

5.3 Implied and adjusting risk aversion

Historically, empirical computations of risk aversion have been a very challenging and mostly academic task. Not so much any more. We can now imply risk aversion directly from clients' positions or ideas, adjust it if necessary and use it to structure the optimal product.

As an illustration let us take the example of two skew investors which we introduced in [1]. On Figure 2 we use f to denote the growth-optimal payoff for Investor 1. In [1] we introduced Investor 2 as a different growth-optimizing investor hinting that we might also look at him as someone who is more risk averse in the wings and less risk averse near ATM. We now know that this is possible because of the investor equivalence principle. To confirm this interpretation numerically all we have to do is to use Investor's 2 profile as F and compute R via the elasticity equation. The result is displayed on Figure 2.B.

In the general case there would be many regions when one investor would be more risk-averse than the other and vice versa. If we want simple risk-aversion relationships, like we have between these two investors, we have to build them. The neat profile for R reveals how this was done.



Imagine now that we want to correct Investor 2 to make sure that he is strictly more risk averse than Investor 1 – not just in the wings but everywhere. For that we just need to adjust the risk-aversion profile so that it lies above 1 which is the value of risk aversion for a growth-optimizing investor. We suggest \tilde{R} (see Figure 2.B) as the alternative. We recompute the optimal profile (see \tilde{F} on Figure 2.C) and observe that there is no longer any additional leverage in the ATM region.

5.4 Rules of thumb

One-parameter families and risk aversion profiles are powerful tools. It would be even better, however, if we could say something about products even when knowing next to nothing about risk preferences. Amazingly enough, this is possible. Note for instance that positive risk aversion means that both global and local extremum points for F and f must coincide. Indeed as we vary x both $F(x)$ and $f(x)$ should be going up or down together for the elasticity $d \ln F / d \ln f$ to remain positive. More can be said about investors who describe their preferences as state-independent, i.e. dependent only on the properties of the investment and not on external factors such as the state of economy. In such cases F is a function of f . This means, for example, that F and f intersect on a bond line, i.e. if $F(x_1) = f(x_1)$ and $F(x_2) = f(x_2)$ then $F(x_1) = F(x_2) = c$, where c is a constant (not necessarily 1).

Because structuring is often constrained by practical considerations (such as the number of traded strikes which we could use for replicating the final product) such rules of thumb provide us with a lot of information. Couple that with additional data like maximum tolerable loss and you might be able to sketch the final optimal structure without detailed knowledge of client's risk-aversion.

6 Summary and outlook

At the heart of structuring we always have optimization which reflects the goals of the client. This is preceded by preparatory steps defining the problem and followed by packaging of the solution into a tradeable product. Together these steps form a backbone of a manufacturing process which we summarize on Figure 3.

The preparatory steps include deciding on the market variable, capturing the relevant market and client views and identifying the goals of the client. This leads us to the key

solution stage where our equations come in: the growth-optimal $b = fm$ and the payoff elasticity equation. After the solution stage we check the quality of the derived product and we may decide to go back and redefine the underlying variable. In [1] we considered a detailed example when this happens. Having satisfied ourselves with the solution we proceed to the final stage of packaging it into a tradeable product.

The success of our businesses and our industry is ultimately determined by the success of our products. Given the seriousness of this fact, we must take steps to ensure a responsible development of our theory. One way of doing that is to look at key stakeholders in the financial industry and theme the development to aid each and every one of them. This brings us to the following summary.

- **Customers – better service**

Client-centric approach is and should remain at the very heart of our framework: in our theory investors are the ultimate beneficiaries by design. We put clients in a position to demand accurate representation of their views in a derivative product as well as to pursue a consistent risk-aversion strategy. Sophisticated clients can use our framework to design their own solutions or at least have an informed idea of what good solutions might look like.

- **Structurers – sharper quantitative tools**

In terms of specific structuring problems, our approach helps with differentiation, integration and extrapolation of customer views (see [1] for details). The ability to claim mathematically accurate fit of clients' views and preferences would constitute a major achievement as well as a selling point for any investment product. In this area there will always be demand for more examples.

- **Researchers – closer to the business**

Implied probability distributions are a common tool of market research. Our framework allows researchers to bring their investment advice closer to the business by formulating it directly in terms of optimal product structures. Because packaging and marketing of the products is normally done by other departments, there is virtually no barrier of entry for researchers.

- **Traders – better hedgeability**

Based on the fundamental laws of learning (Bayes Theorem), our approach would tend to result in products with exposure only in those scenarios for which at least some information (and therefore some trading) is available on the market. In other words, by asking for market data, $m()$, and insisting that clients' views, $b()$, are based on a learning process on top of market view, the theory pushes the customers towards taking tradable and therefore hedgeable risks. Upgrade of the theory to include existing positions (re-structuring as opposed to structuring) is probably the next logical development in this area. Under the assumption of zero bid-offer spreads this generalization is trivial – the challenge is to include friction.

- **Quants – new profession**

The historical approach of taking financial products for granted and then trying to stabilize them through modelling does not make sense. Our theory provides a unified approach where product structures are borne side-by-side with the relevant

implied probabilities. Quantitative structuring might become a new client-facing profession for many experienced modelling quants. The classic quant profession still benefits from new tools (e.g. model risk assessment – see [1] for more details).

- **Risk Managers – better predictability**

The combined effect of better hedgeability (see above), the logical structure of investments and the ability to monitor the creation and progression of trading ideas through the clear manufacturing process (see Figure 3) should add to predictability of businesses. Product validation considered in this paper can provide further aid to the standard trade validation processes. More examples of product validation would be helpful.

- **Regulators – better transparency and control**

Looking at the established industries we see that safety is created as a cumulative effect of various control measures throughout the entire manufacturing process. Positioning product innovation as a clear manufacturing process rather than a form of art should add transparency and make it much easier to police.

- **Economists – additional tools**

The wider field of economics might benefit from new paradigms such as the investor equivalence principle as well as specific tools such as the ability to compute risk aversion profiles from trading positions (payoff elasticity equation).

- **Economy at large – smaller dislocations**

Unforeseen economic events will be happening no matter what. Having accepted that, what we should do is to minimize the consequences of such events. One type of such consequences are contractual dislocations – situations when one group of people ends up owning another group of people astronomical payouts as a result of a completely unforeseen event. Liquidity problems, accusations of greed, gambling, unfair treatment and so on are commonplace symptoms of such dislocations. Having resolved the payout extrapolation problem we put ourselves in a good position to avoid contractual dislocations (see [2] for additional discussion). Further research is required.

Although the current level of development varies among the above themes, we see that our simple framework already goes quite a long way.

7 Appendix: Rationality and Logic

Apart from rationality, as captured by the expected utility approach, there is another technical aspect of decision making which is extremely important but which is often taken for granted. This aspect is the mathematical foundation of logic itself.

The simplest and perhaps the most widely known example of logical framework is given by the Boolean algebra which uses True=1 and False=0 to describe logical statements. Boolean algebra has proven itself as an extremely powerful tool underpinning, among other things, all computer-based calculations. Amazingly enough, despite of all of its successes, Boolean approach leaves a substantial room for improvement.

Indeed, what should we do with statements for which we cannot say if they are true or false? Imagine for example a statement for which we only have a measure of how often or how likely it is to be true. It turns out that the Boolean approach can be very easily generalized to handle such cases [5]. All we have to do is to replace True and False by numbers between zero and one and, of course, we have to generalize the rules for manipulating these numbers. The resulting logical system turns out to coincide with the standard probability theory. In particular, the product rule which we use to compute joint probability

$$p(A, B) = p(A)p(B|A) = p(B)p(A|B) \quad (27)$$

arises as a generalization of the Boolean product which we use to determine the truth value of the joint statement (A and B). The only conceptual difference is that statements can now have varying degree of information about one another which is handled by the concept of conditioning (as in conditional probabilities). The last equality in (27), i.e. the Bayes theorem, emerges as a logical consistency requirement which comes from swapping the order in which we look at A and B .

This intimate connection of the Bayes theorem with logic justifies the emphasis we make on logical investors. Indeed, violations of the Bayes theorem do not just go against basic probability theory. Such violations may also be a symptom of much deeper logical inconsistencies which can put investors on a path of potential contradiction with all sorts of mathematical results – from the present day and all the way back to Aristotle. At the moment, the simplest known way of avoiding this is to make sure that the advice which we provide to our investors is not just rational (as per expected utility theory) but is also logical. In other words, we need to make sure that our investors learn new information, i.e. update their views, in line with the Bayes theorem. Of course, the utility theory itself is already based on logic. We can therefore expect, at least in some cases, for logical behaviour to be already included in rationality. This is what allowed us to use such simple derivations.

References

- [1] Soklakov, A., “Deriving Derivatives”, submitted to RISK, (April 2013). arXiv:1304.7533.
- [2] Soklakov, A., “Bayesian lessons for payout structuring”, RISK, Sept. (2011), 115-119. arXiv:1106.2882.
- [3] von Neumann, J. and Morgenstern, O., “Theory of Games and Economic Behaviour”, Princeton (1944, 2nd ed. 1947, 3rd ed. 1953).
- [4] Kelly J L Jr, “A New Interpretation of Information Rate”, Bell System Technical Journal, 917-26 (1956).
- [5] Jaynes, E. T., “Probability Theory, The Logic of Science” (2003).
- [6] Shimko, D., “A tail of two distributions”, RISK, Sept. (1994), 123-130.